W. B. DANIELS AND CHARLES S. SMITH

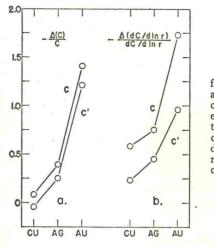


FIG. 2. Closure failures Δ expressed as fractions of the corresponding total experimental quantity. (a) Elastic shear constants; (b) hydrostatic strain derivatives of shear constants.

corrections which have been subtracted from experimental values. Since it is felt that this homologous series of metals must conform to a common model of their elastic stiffnesses and strain derivatives, we include copper in our conclusion, which is that the closure failures reflect a large, real contribution to the shear constants which is not included in the conventional theory outlined above.

We suggest furthermore that the closure failures must be assigned to many-body, noncentral, shortrange interaction between metal ion cores. The absence of such an interaction is a major assumption in the conventional theory and the interaction seems to be the only way in which to account for these large discrepancies between theory and experiment for the shear constants. The ratios $-\Delta(\Omega dC/d\ln r)/\Delta(\Omega C)$ and $-\Delta(\Omega dC'/d \ln r)/\Delta(\Omega C')$ are indicative of the range of the interaction; the large values of these ratios occurring in the present results indicates that the noncentral terms are of short range indeed. It will be noted that the values of the ratios are in most cases larger than the value of p which characterizes the range of the radial part of the interaction. Further, the smoothness of the variation of the closure term from copper to gold for each constant and each strain derivative corresponds with the increasing amount of ion-core overlap and hence of the importance of the noncentral interaction in this sequence. There appears to be no theory available for the noncentral part of the many-body interaction between ion cores which has been suggested here and no a priori reason for or against the negative sign of the stiffness contribution which is found.

DISCUSSION

In this section we point out and discuss further the detailed assumptions involved in the analysis which was presented in the last section.

The experimental elastic constant values which have been used for the interpretation are those for the temperature of 0°K. These values are amply known, and their use enables us to avoid the difficult theoretical

question of the temperature dependence of the elast constants, and in addition we avoid the minor post of the adiabatic-isothermal correction to the bur modulus. On the other hand, the experimental values of the hydrostatic strain derivatives which have here used are of necessity those for room temperature. 1 analysis is somewhat inconsistent in this respect there fore, but we do not feel that the point is important we expect a smaller temperature correction for the hydrostatic strain derivatives than for the elast constants, which itself is less than 10%. The number direct justification for this expectation may be obtained from the results of Bridgman on the pressure deperence of the bulk modulus. Bridgman's experiments have been carried out at two temperatures, 30°C and 75% and it is the coefficient b, in our notation, which relevant. Inspection of Bridgman's tabulation4 for some forty metals shows that there is practically no char of b in this temperature range for most metals. Further more, for those metals for which there is a significant change, the sign is as often positive as negative. We feel it quite probable that the hydrostatic strain derivation tives of the shear constants will also show only a small temperature dependence.

We have used also the hydrostatic strain derivative of the *adiabatic* bulk modulus in our interpretation. A direct but approximate evaluation of the pressure derivative of $B_s - B_T$ can be made from thermodynamic and available experimental data for Cu; the result that the pressure derivatives of B_s and B_T differ by less than 2%. We have preferred to avoid the uncertainty involved in this correction by using the module which is directly determined in the pulse-echo method

It has already been emphasized that the contritions of the long-range terms in the energy to the ela stiffnesses and their hydrostatic strain derivatives at small. In the interpretation they may almost be garded as corrections but some further discussion worthwhile. In the conventional theory, as it has be used in the last section, one term in the energy of " crystal is commonly omitted in part. This term is " energy of the lowest electronic state of the valet electrons, which will be denoted by E_0 . Physically can be represented by the expression²⁴ $ar^{-3}-br^{-1}$ which the terms represent respectively the kine and potential energy associated with the state. In the conventional theory for the shear constants, E_0 approximately approximately the shear constants approximately to a good approximation as the Coulomb stiffness ΩC_{lr} and $\Omega C_{lr'}$ as used here.

The lowest state energy contribution to the base modulus and its hydrostatic strain derivative has been ignored entirely, however, and in justification of the step the magnitudes of the derivatives E_0'' and E_0'' must be considered. The first derivative of E_0 is here but does not enter in this analysis at all because the equation of equilibrium has been invoked implicitly by

²⁴ N. F. Mott and H. Jones, *Properties of Metals and Alloc* (Oxford University Press, London, 1936), p. 80.

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